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Polarized Communities meet Densest Subgraph: Efficient and Effective Polarization Detection in Signed Networks

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Overview

- Problem & Motivation. The **2-Polarized-Communities (2PC)** task (Bonchi et al., 2019) seeks two disjoint node sets in a *signed network* that are internally cohesive (mostly positive intra-group edges) and mutually antagonistic (mostly negative inter-group edges), while keeping their total size relatively small. Prior work has neither examined the implicitly optimized density measure by 2PC nor leveraged the rich algorithmic toolbox developed for the **Densest Subgraph (DS)** problem.

- Key Idea. We show that 2PC can be viewed as a densest-subgraph–like optimization problem on signed graphs, where the optimized density is defined via a signed degree measure, dubbed **net degree balance**, that rewards compliant edges and penalizes noncompliant ones.

- Algorithm & Evaluation. Building on this bridge, we propose Greedy-2PC, a **greedy peeling algorithm**, seeded by a spectral relaxation. We evaluate Greedy-2PC on real-world and synthetic signed networks, conducting a comparative evaluation against Pivot (Bansal et al., 2004), BNC (Chiang et al., 2012), SPONGE (Cucuringu et al., 2019), SSSNet (He et al., 2022), TIMBAL (Ordozgoiti et al., 2020), EIGEN (Bonchi et al., 2019), Neural2PC (Gullo et al., 2024), and RH (Chen et al., 2024), .

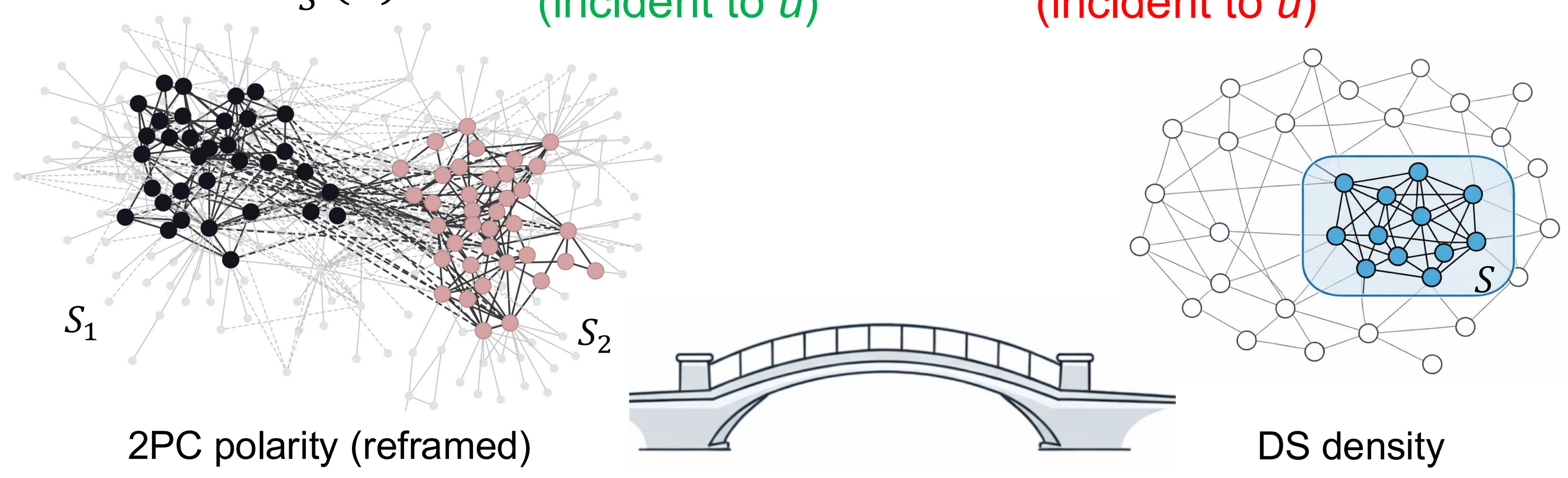
Bridging 2PC and DS

2PC. Given a signed graph $G = (V, E^+, E^-)$, find $S_1, S_2 \subseteq V$, that **maximizes**:

$$p(S_1, S_2; G) = \frac{\sum_{i\in\{1,2\}}(|E^+(S_i)| - |E^-(S_i)|) + |E^-(S_1,S_2)| - |E^+(S_1,S_2)|}{|S_1 \cup S_2|}$$

Key concept. *Net Degree Balance:* For any node u, its net degree balance w.r.t. a pair of polarized communities $S = \{S_1, S_2\}$ is

$$d_S^{\pm}(u) = \text{\#compliant edges (incident to } u) - \text{\#noncompliant edges (incident to } u)$$



2PC polarity (reframed)

$$p(S_1, S_2; G) = \frac{\sum_{u\in S_1\cup S_2} d_S^{\pm}(u)}{|S_1 \cup S_2|}$$

DS density

$$\frac{\sum_{v\in S} d_S(v)}{|S|}$$

The Greedy-2PC Algorithm

Algorithm 2 Greedy-2PC

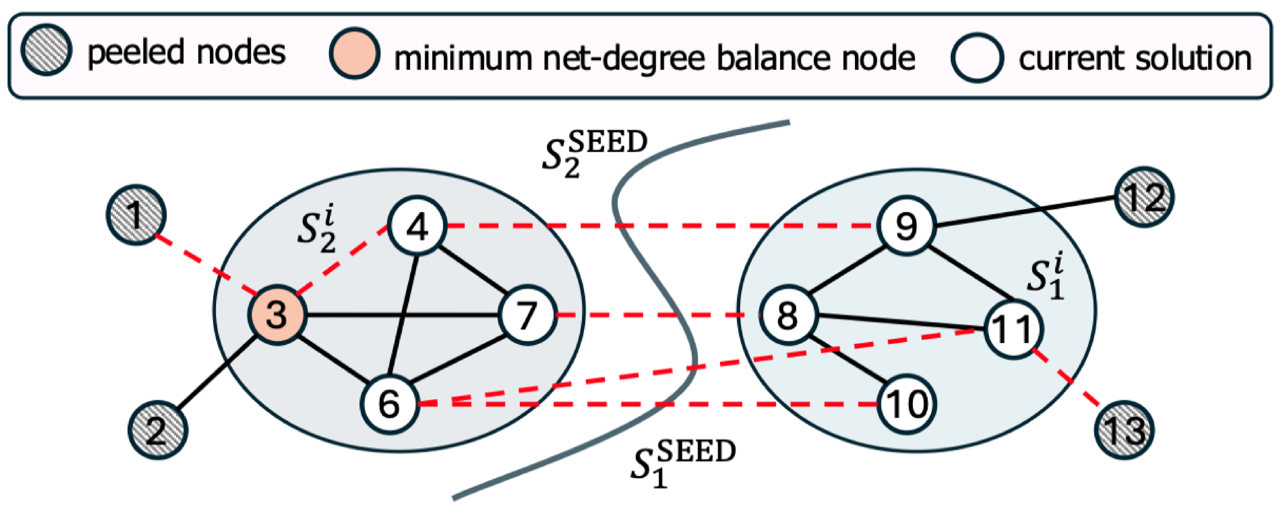
Input: Signed graph $G = (V, E^+, E^-)$
Output: A pair $\widehat{S} = \{S_1, S_2\}$ of polarized communities

```
S^SEED ← EIGEN-FULL(G)   {Algorithm 1}
Ŝ ← S^SEED,  S^n ← S^SEED,  i ← n
while |S_1^i ∪ S_2^i| > 1 do
    u = arg min_{v ∈ S_1^i ∪ S_2^i} d^±_{S^i}(v)
    remove u from S_1^i or S_2^i in S^i
    if p(S_1^i, S_2^i; G) > p(Ŝ_1, Ŝ_2; G) then
        Ŝ ← S^i
    end if
    i ← i − 1
end while
return Ŝ
```

Algorithm 1 EIGEN-FULL

Input: Signed graph $G = (V, E^+, E^-)$
Output: A pair $S = \{S_1, S_2\}$ of polarized communities

```
Compute z*, the eigenvector corresponding to the
  largest eigenvalue λ_1 of the signed adjacency matrix A of G
S_1 = {u ∈ V : z*_u ≥ 0}, S_2 = {u ∈ V : z*_u < 0}
```



Key Benefits of Greedy-2PC

Highly efficient: Greedy-2PC runs in $O(|V| + |E|)$ time.

Effective in practice: It consistently outperforms SOTA methods for 2PC.

Theoretical (additive) guarantees: Under condition $S_1^* \subseteq S_1^{seed}, S_2^* \subseteq S_2^{seed}$, $p(\widehat{S}_1, \widehat{S}_2; G) \geq OPT - c$, where c is a term related to the (maximum) #noncompliant edges of the peeled nodes.

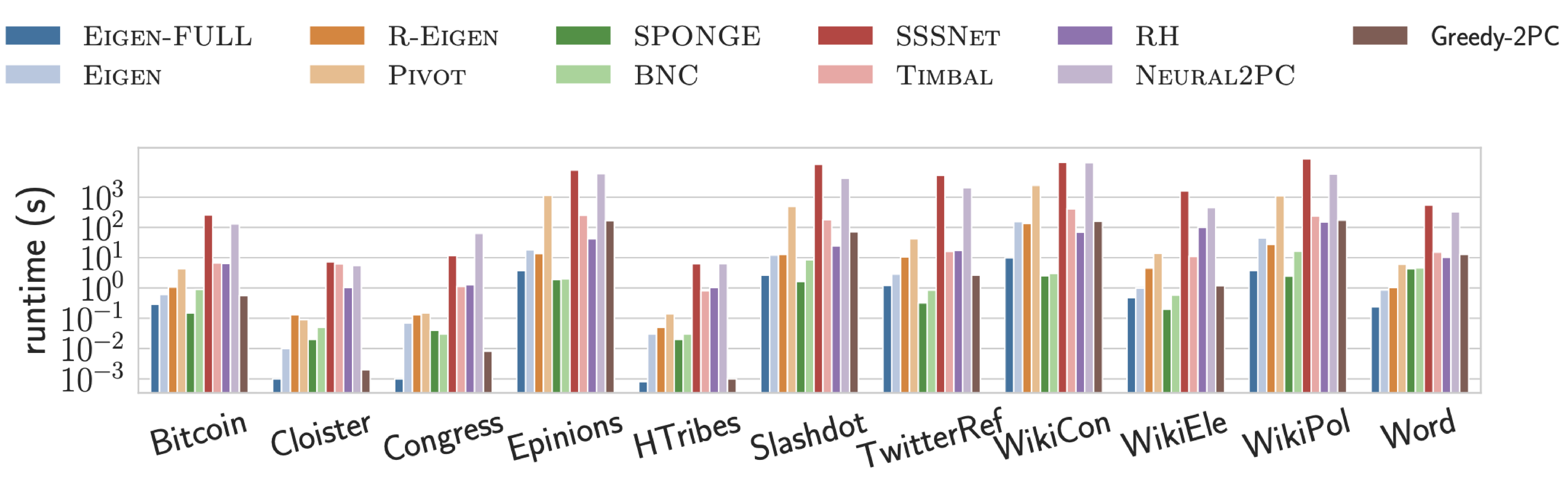
Simple to implement.

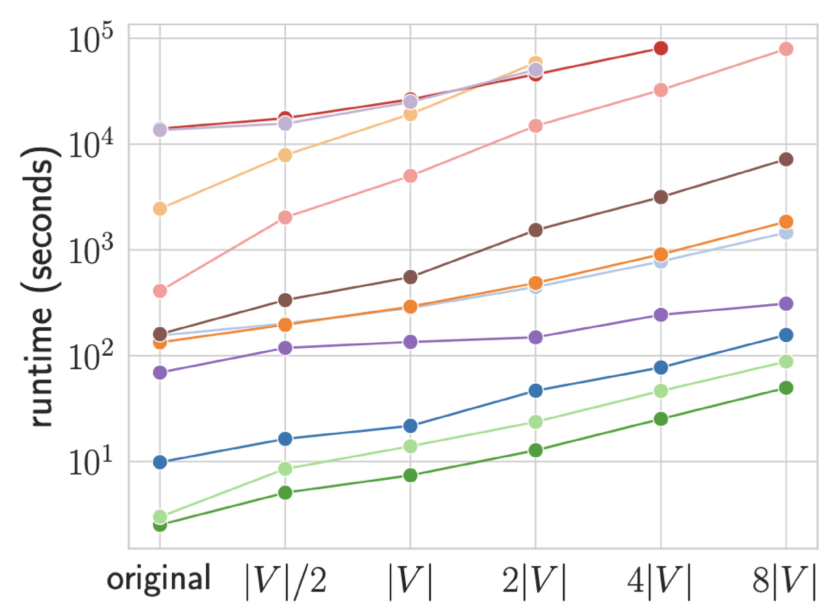
Results

Comparative evaluation. Greedy-2PC outperforms competing methods on 11 real-world datasets in polarity and agreement ratio, and on synthetic graphs generated with a *modified Signed Stochastic Block Model* (with ground-truth communities) in F1 and polarity across noise levels η, while remaining highly efficient and scalable on large networks (poster: we report only the strongest competitors for effectiveness; full tables are in the paper).

method	criteria	*Bitcoin*	*Cloister*	*Congress*	*Epinions*	*HTribes*	*Slashdot*	*TwitterRef*	*WikiCon*	*WikiEle*	*WikiPol*	*Word*
EIGEN-FULL	pol.	6.23	6.11	4.37	8.89	5.50	8.08	39.03	28.78	19.58	8.57	9.87
	a.r.	0.93	0.72	0.96	0.91	0.88	0.83	0.92	0.91	0.85	0.91	0.76
EIGEN	pol.	29.52	**7.45**	6.58	128.72	**6.18**	79.7	174.1	175.65	71.73	88.44	24.02
	a.r.	0.95	0.94	0.98	0.95	1.00	0.99	0.99	0.95	0.93	0.96	0.98
RH	pol.	29.30	7.39	6.50	170.54	**6.18**	82.39	174.40	190.52	72.64	89.50	24.28
	a.r.	0.96	0.91	0.98	1.00	1.00	0.99	1.00	0.96	0.92	0.97	0.97
NEURAL2PC	pol.	30.28	**7.45**	6.64	171.1	**6.18**	82.25	174.35	187.29	72.17	88.89	24.32
	a.r.	0.95	0.94	0.98	1.00	1.00	0.99	0.99	0.95	0.92	0.96	0.97
Greedy-2PC	pol.	**30.57**	**7.45**	**6.70**	**171.17**	**6.18**	**82.80**	**174.66**	**196.73**	**72.79**	**90.07**	**25.03**
	a.r.	0.96	0.94	1.00	1.00	1.00	0.99	1.00	0.97	0.92	0.97	0.98

method	criteria	η = 0	0.1	0.2	0.3	0.4	0.5	0.6
EIGEN-FULL	F_1	.333	.334	.333	.333	.334	.333	.264
	pol.	39.80	44.86	44.55	40.51	37.94	30.23	28.82
EIGEN	F_1	**1.0**	.998	.998	.998	**.995**	.972	.307
	pol.	**199**	168.04	140.31	110.5	**81.44**	50.02	35.52
RH	F_1	**1.0**	.99	**1.0**	**1.0**	**.995**	0.81	0.25
	pol.	**199**	167.89	**140.65**	**110.69**	**81.44**	45.83	35.76
NEURAL2PC	F_1	**1.0**	**1.0**	**1.0**	**1.0**	**.995**	**.997**	.341
	pol.	**199**	**168.62**	**140.65**	**110.69**	**81.44**	50.27	36.16
Greedy-2PC	F_1	**1.0**	**1.0**	**1.0**	**1.0**	**.995**	.988	.293
	pol.	**199**	**168.62**	**140.65**	**110.69**	**81.44**	**50.28**	**38.1**





Scan the QR code to access the Greedy-2PC GitHub repository

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